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Also solved by O. S. ADAMS, C. N. SCHMALL, PAUL CAPRON, and HORACE OLSON.

MECHANICS.

324. Proposed by H. S. UHLER, Yale University.

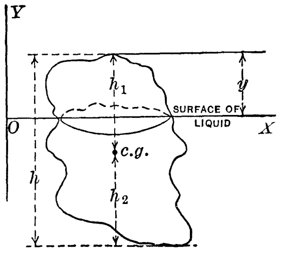
A rigid body of any shape is at rest in a neutral liquid which is also at rest and has an indefinitely great volume. The body is so situated that the free surface of the liquid is tangent to it at its highest point (or points). All the space above the liquid is filled with a neutral stagnant fluid whose density is not greater than the density of the liquid. Show that the work done in raising (pure translation) the body very slowly until the interface of the two fluids is tangent to it at its lowest point (or points) is expressible by the formula $mgh - gV(\rho_1 h_1 + \rho_2 h_2)$, where $m \equiv$ mass of body, $V \equiv$ volume of body, $\rho_1 \equiv$ mean density of lower medium, $\rho_2 \equiv$ density of upper medium, $h_1 \equiv$ distance of center of mass of the displaced liquid below the free surface in the initial position of the body, $h_2 \equiv$ elevation of center of mass of displaced fluid above the interface in final position of the body and $h = h_1 + h_2$. (Neglect surface-tension, etc.)

SOLUTION BY J. B. EPPES, Annapolis, Md.

Take the axes indicated in the figure. Let the highest point of the body at any time be a distance y above the surface of the liquid, and let A be area of the cross-section of the body made by the surface of the liquid.

Then the mass of the fluid displaced is $\rho_2 \int_0^y A dy$ and the mass of the liquid displaced is $\rho_1 \left(V - \int_0^y A dy \right)$.

Hence, the downward force is $g \left[m - \rho_2 \int_0^y A dy - \rho_1 \left(V - \int_0^y A dy \right) \right]$. Then, the total work in raising the body, is



$$\int_0^h \left[mg - \rho_2 g \int_0^y A dy - \rho_1 g \left(V - \int_0^y A dy \right) \right] dy = mgh$$

$$- \rho_2 g \int_0^h dy \int_0^y A dy - \rho_1 g V h + \rho_1 g \int_0^h dy \int_0^y A dy.$$

Now assume A as a function of y to be of the form

$$A \equiv B + Cy + Dy^2 + \cdots.$$

Then,

$$\int_0^h dy \int_0^y A dy = \int_0^h \left(By + \frac{Cy^2}{2} + \frac{Dy^3}{3} + \cdots \right) dy = \frac{Bh^2}{2} + \frac{Ch^3}{2 \cdot 3} + \frac{Dh^4}{3 \cdot 4} + \cdots$$

$$= \left[Bh^2 + \frac{Ch^3}{2} + \frac{Dh^4}{3} + \cdots \right] - \left[\frac{Bh^2}{2} + \frac{Ch^3}{3} + \frac{Dh^4}{4} + \cdots \right]$$

$$= h \int_0^h A dy - \int_0^h yA dy = hV - Vh_1 = Vh_2.$$

Hence, work $= mgh - \rho_2 g V h_2 - \rho_1 g V h + \rho_1 g V h_2 = mgh - gV(\rho_1 h_1 + \rho_2 h_2)$.

325. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

The lever of a testing machine is c feet long, and is poised on a knife edge a feet from one end and b feet from the other, and in a horizontal line above which the beam is symmetrical. The beam is m inches deep at the knife edge and tapers uniformly to a depth of n inches at each end; the width of the beam is the same throughout its length. Find the distance of the center of gravity of the beam from the knife edge.